

Effective Field Theory, Renormalizability and Extra Dimensions

Bei Jia^{1,2}

¹*Institute of Modern Physics, Chinese Academy of Sciences, P.O.Box 31 Lanzhou, 730000, China*

²*Graduate University of Chinese Academy of Sciences, Beijing, 100080, China*

Abstract

We discuss in this paper two ways of defining the concept of “effective field theory”: effective field theory defined by low energy effectiveness and effective field theory defined by 4D effectiveness out of higher dimensions. We argue that these two views are actually equivalent, that effective field theories at low energy can in fact be regarded as field theories of higher dimensions confined on a 4D spacetime. We examine this idea through comparing two different regularization schemes: Momentum Cutoff and Dimensional Regularization, and through analyzing how fields can be localized on branes.

1 Introduction

Quantum field theories are nowadays usually described as “effective field theories” (EFT) which are only valid below some energy cutoffs [1~6]. It is generally believed that we may need a completely new theory to describe the physics beyond those energy levels, such as string theory. Also, from an EFT point of view, renormalizability, which used to be a standard benchmark for acceptable quantum field theories not very long ago, is no longer relevant. On the other hand, since the early works of Kaluza and Klein [7], the concept of extra dimensions has been broadly consumed by physicists. Now the interests have been shifted from the traditional Kaluza-Klein type to the so called “brane world” picture, in which some fields (like SM fields) are localized at a brane while other fields (such as gravitation) can propagate in more dimensions. Numerous models of this kind have been

proposed for different purposes, such as large extra dimensions models like Arkani-Hamed, Dimopoulos and Dvali (ADD) scenario [8, 9] and warped extra dimensions models like Randall-Sundrem (RS) models [10, 11].

One might wonder whether the low-energy EFT has any relation with the 4D EFT. This is the main topic of this paper and we are trying to answer this question, at least providing a possible clue. We shall first compare two different regularization schemes in renormalization: Momentum Cutoff and Dimensional Regularization, in order to look in to the relationship between effective field theory with an energy cutoff and effective field theory in four dimensions. Then in section 3 we move on to discuss the localization of fields on a 3-brane, in which we will see the non-zero modes of higher dimensional fields can leave the brane if they reach a high energy level. Thus the concept of low energy effectiveness and 4D effectiveness can also be related in this way.

2 Regularization and Renormalization

The formal process of renormalization falls into two parts: regularization and subtraction. Basically we first “regulate” divergence in the momentum integral, and then we bring in some “counter-terms” to remove the divergence [12]. In renormalization we are dealing with momentum integrals like

$$I = \int_0^\infty d^4k F(k) \quad (1)$$

Usually there are two common ways of regularization: Momentum Cutoff (MC) and Dimensional Regularization (DR). In the scheme of Momentum Cutoff, we change the upper limit of the integral in Equation (1) from ∞ to a momentum cutoff (usually very large) Λ

$$I \rightarrow I_\Lambda = \int_0^\Lambda d^4k F(k) \quad (2)$$

where I_Λ is convergent, and becomes I in the limit of $\Lambda \rightarrow \infty$. Generally this modified integral can be calculated as

$$I_\Lambda = A(\Lambda) + B + C\left(\frac{1}{\Lambda}\right) \quad (3)$$

We can see when we perform the limit of $\Lambda \rightarrow \infty$, we will have $C(1/\Lambda) \rightarrow 0$,

B remains the same and $A(\Lambda) \rightarrow \infty$, which represents the divergence of the original integral of I . This momentum cutoff Λ can actually be described as the energy level under which the field theory we are using here is effective.

On the other hand, traditionally we start the Dimensional Regularization scheme with the modification of Equation (1)

$$I \rightarrow I_D = \int_0^\infty d^D k F(k) \quad (4)$$

which means we are shifting the integral from a four dimensional one to a D dimensional one. It is important to notice that traditionally we start with a four dimensional field theory, and only shift to higher dimensions in the process of DR. We will come back to this later. By defining $\epsilon = 4 - D$, we can rewrite the integral as

$$I_D = A(\epsilon) + B + C\left(\frac{1}{\epsilon}\right) \quad (5)$$

similarly we will have under the limit of $D \rightarrow 4$ ($\epsilon \rightarrow 0$), that $A(\epsilon) \rightarrow 0$, B remains the same and $C(1/\epsilon) \rightarrow \infty$. Again we “parameterized” the divergence of the original integral, in order to remove it in the next step of renormalization.

Those are the formal ways to perform MC and DR. We would like to point out that rather than starting with a four dimensional field theory and then perform DR in the process of renormalization, we should begin with a D dimensional field theory. By this we mean we should not wait until we meet the infinity of the momentum integral and then artificially increase the dimensions. Rather, we should start our theory in a D dimensional space-time and then shift it to 4D in order to have an effective theory. The great usefulness of DR then can be interpreted as following: in order to have a finite effective theory we have to “confine” our starting theory (which is D dimensional) to 4D spacetime. This point of view suggests that the divergence of the usual 4D field theory may be regarded as the effect of higher dimensions.

Let us consider a D dimensional scalar field $\phi(x_\mu, y_n)$ in a D dimensional Minkowski spacetime, where x_μ ($\mu = 0, 1, 2, 3$) denotes the four dimensional coordinates, while y_n ($n = 4, 5, \dots, D - 1$) are the coordinates of extra dimensions. We consider the action

$$S = \int d^4 x d^{D-4} y \left[\frac{1}{2} (\partial_A \phi(x_\mu, y_n))^2 - \frac{m^2}{2} \phi^2(x_\mu, y_n) - \frac{\lambda}{4!} \phi^4(x_\mu, y_n) \right] \quad (6)$$

where A denotes all the D dimensional coordinates. In order to get the four dimensional action, we formally integrate over y_n

$$S = \int d^4x \left[\frac{1}{2} Z_1 (\partial_\mu \phi_{(4)}(x_\mu))^2 - \frac{m^2}{2} Z_2 \phi_{(4)}^2(x_\mu) - \frac{\lambda}{4!} Z_3 \phi_{(4)}^4(x_\mu) \right] \quad (7)$$

where Z_1 , Z_2 and Z_3 are from the integration over y_n , and $\phi_{(4)}(x_\mu)$ is the result of the integration, which is a four dimensional scalar field. We can see the four dimensional effective lagrangian is

$$\mathcal{L}_{eff} = \frac{1}{2} Z_1 (\partial_\mu \phi_{(4)}(x_\mu))^2 - \frac{m^2}{2} Z_2 \phi_{(4)}^2(x_\mu) - \frac{\lambda}{4!} Z_3 \phi_{(4)}^4(x_\mu) \quad (8)$$

This reminds us the traditional argument about the renormalization of four dimensional ϕ^4 theory, in which we have the “rescaled lagrangian”

$$\mathcal{L} = \frac{1}{2} Z (\partial_\mu \phi_r(x_\mu))^2 - \frac{m_0^2}{2} Z \phi_r^2(x_\mu) - \frac{\lambda_0}{4!} Z^2 \phi_r^4(x_\mu) \quad (9)$$

where $\phi^2 = Z \phi_r^2$, and m_0 and λ_0 are the “bare parameters”. We can see that, instead of referring the divergence to the “bare parameters”, we may say the divergency in the four dimensional effective lagrangian might come from Z_1 , Z_2 and Z_3 , which are the effect of the extra dimensions.

There are many problems about constructing field theories in higher dimensions [13] and we will not go into this here. We will focus on the concept of renormalizability. It is important to notice that in [14] the nonperturbative renormalizability of higher dimensional gauge field theories is discussed using the functional RG, which reaches a similar idea with ours. We should mention that the idea of effective field theory comes from the original work of Wilson about renormalization group [1]. Traditionally a field theory is called renormalizable if the divergent terms in Equations (3) and (5) can be practically removed by some sort of subtraction. We think that this renormalizability is only valid in the view of an effective theory [14, 15] — when we reach the energy cutoff, our traditional four dimensional theories lost their effectiveness. Maybe we need brand new physics like string theory, or we can argue that the loss of effectiveness is the result of both higher energy level and the effect of higher dimensions. When we reaches a rather high energy level, the extra dimensions show up and particles which are used to be confined in 4D spacetime may escape to those extra dimensions, and the

4D effective description of field theories is no longer valid. This leads to the main topic of the following section, which is the relationship between localization of fields and their 4D effectiveness.

3 Localization of Fields and Energy Cutoff

There are two views towards fields in extra dimensions: fields can live on a brane and do not have any freedom along the extra dimensions; or higher dimensional fields can be localized on the brane through some kind of mechanism. In the second view we need to find out the mechanism about how fields are localized. The idea of localizing fields to a topological defect originated since the early 1980s [16~19], and serves lots of different purpose such as symmetry breakings [20, 21], fermion mass hierarchy [22, 23] and proton decay [23]. Here we will follow the method used in [24] of how to localize fermions on a brane.

In the simple model in [24] it is assumed that there is only one single extra dimension parameterized by z . There is a 5D real scalar field φ whose action is

$$S_\varphi = \int d^4x dz \left[\frac{1}{2}(\partial_A \varphi)^2 - V(\varphi) \right] \quad (10)$$

where A denotes all five coordinates. The scalar potential $V(\varphi)$ has a double-well shape with two degenerate minima at $\varphi = \pm v$. There exists a classical solution $\varphi_c(z)$ which is dependent on z only and can serve as a domain wall separating two classical vacua. Introducing the Yukawa type interaction between fermions and the scalar field φ , the five-dimensional action for fermions can be written as

$$S_\Psi = \int d^4x dz (i \bar{\Psi} \Gamma^A \partial_A \Psi - \lambda \bar{\Psi} \Psi) \quad (11)$$

where Ψ is the fermion field and $\Gamma^\mu = \gamma^\mu$, $\mu = 0, 1, 2, 3$; $\Gamma^z = -i\gamma^5$ with γ^μ and γ^5 being the usual Dirac matrices. Then the corresponding 5D Dirac equation is

$$i\Gamma^A \partial_A \Psi - \lambda \varphi_c(z) \Psi = 0 \quad (12)$$

There exists a four-dimensional left-handed zero mode

$$\Psi_0 = e^{-\int_0^z dz' \lambda \varphi_c(z')} \psi_L(p) \quad (13)$$

where $\psi_L(p)$ is the usual solution of the 4D Weyl equation. We can see the zero mode is localized near $z = 0$, i.e., at the domain wall. It is this zero mode which can be described as our usual four-dimensional matter, who can acquire small masses through some other mechanisms. This can also be interpreted as the four-dimensional effectiveness of an effective field theory, since interactions between zero modes can only produce zero modes again at low energies.

There are other massive modes of the 4D fermions. The five-dimensional fermions have a mass in the scalar field vacua $M = \lambda v$. Besides the zero mode, 4D fermion field have a continuum part of the spectrum starting at M , which correspond to 5D fermions which are not bound to the domain wall. Zero modes interacting at high energies will produce these continuum modes, and the effect of the higher dimensional part of the field will show up. This is where the four-dimensional description of the theory loses its effectiveness. Therefore M can be regarded as an energy level under which our theory is effectively four-dimensional, and the two usual meanings of the term “effectiveness” are related here.

4 Conclusion

The relationship between low-energy effectiveness and four-dimensional effectiveness can be rather complicated, and we only provide a first sight on this question. By ignoring lots of important problems, we try to focus on the main topic, and argue that this relationship between the two effectiveness does exist, at least technically. Further work need to be done in order to investigate the specific relationship between energy cutoff and localization of higher dimensional fields. It is also crucial to clarify the concept of renormalizaion in this relation about effective field theory. An important work about this question can be found in [14].

References

- [1] K. G. Wilson and J. B. Kogut, Phys. Rept. **12**, 75 (1974).
- [2] H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin, 1984.
- [3] L. J. Hall, Nucl. Phys. B **178**, 75 (1981).
- [4] S. Weinberg, Phys. Lett. B **91**, 51 (1980).

- [5] E. Witten, Nucl. Phys. B **122**, 109 (1977).
- [6] I. Z. Rothstein, [hep-ph/0308266v2].
- [7] T. Kaluza, Sitzungsber. Press. Akad. Wiss. Berlin (Math. Phys.) **1921**, 966 (1921); O. Klein, Z. Phys. **37**, 895 (1926) [Surveys High Energ. Phys. **5**, 241 (1986)].
- [8] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [hep-ph/9803315].
- [9] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [hep-ph/9804398].
- [10] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].
- [11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].
- [12] A. E. Blechman, *Renormalization: Our Greatly Misunderstood Friend*, <http://www.pha.jhu.edu/~blechman/papers/renormalization/>
- [13] D. I. Kazakov, [hep-th/0311211].
- [14] H. Gies, Phys. Rev. D **68**, 085015 (2003) [hep-th/0305208].
- [15] R. Rattazzi, [hep-ph/0607055].
- [16] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 139 (1983); Phys. Lett. B **125**, 136 (1983).
- [17] M. Visser, Phys. Lett. B **159**, 22 (1985) [hep-th/9910093].
- [18] K. Akama, Lect. Notes Phys. **176**, 267 (1982) [hep-th/0001113].
- [19] E. Weinberg, Phys. Rev. D **24**, 2669 (1981).
- [20] G. R. Dvali and M. A. Shifman, Nucl. Phys. B **504**, 127 (1997) [hep-th/9611213].
- [21] N. Arkani-Hamed and S. Dimopoulos, [hep-ph/9811353].
- [22] D. B. Kaplan and M. Schmaltz, Phys. Lett. B **368**, 44 (1996).
- [23] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000) [hep-ph/9903417].
- [24] V. A. Rubakov, Phys. Usp. **44**, 871 (2001) [Usp. Fiz. Nauk **171**, 913 (2001) [hep-ph/0104152].